Quantum Physics 141

Problem Set 1

National Institute of Physics

(Dated: August 24, 2018)

Deadline: 5 September 2018

I. FOURIER BASIS [10 pts]

Let $\langle x|k\rangle = e^{ikx}/(2\pi)^{1/2}$ with $\{|k\rangle\}$ and $\{|x\rangle\}$ complete bases sets so that

$$\mathbb{1} = \int dk\, |k\rangle \langle k| = \int dx\, |x\rangle \langle x| \,, \tag{1}$$

and $\langle k|k'\rangle = \delta(k-k')$ and $\langle x|x'\rangle = \delta(x-x')$. Prove that the functions $f(k) = \langle k|\psi\rangle$ and $g(x) = \langle x|\psi\rangle$ are Fourier transforms of each other.

II. WAVEPACKET [50 pts]

At some initial time $t = 0$ the wavefunction of a free $\big(\hat{H} = -\hbar^2 D_x^2/(2m)\big)$ particle of mass m is

$$\Psi(x,0) = \frac{e^{-(x-x_0)^2/(4\sigma^2)}}{(2\pi\sigma^2)^{1/4}}. \tag{2}$$

For this problem you will need to evaluate Gaussian integrals and/or be familiar with gamma functions.

10 pts Verify that this wavefunction is normalized and calculate the uncertainty in measuring the position σ_x^2 at $t = 0$. If σ^2 is finite, the wavefunction is said to be localized.

20 pts The set of plane waves $\{e^{ikx}/\sqrt{2\pi}\}$ are solutions to the eigenvalue equation $\hat{H}\psi(x) = E\psi(x)$ with the energy eigenvalue E and wavevector k related by $E = \hbar^2 k^2/(2m)$. Obtain the full time dependence of the wavefunction $\Psi(x,t)$. *Hint*: Decompose the initial wavefunction in the Fourier basis (basis of plane waves) and evaluate the resulting integral.

20 pts Prove that the wavefunction (a Gaussian wavepacket) disperses (spreads) in time by calculating the σ_x^2 as a function of time t.

Although plane waves are stationary states of the Hamiltonian $\hat{H}$ a linear combination or integrated combination of several plane waves may not be stationary.