Quantum Physics 141

Problem Set 2

National Institute of Physics

(Dated: September 28, 2018)

Deadline: 9 October 2018

I. ANGULAR MOMENTUM CONSERVATION [20 pts]

Let $V(\mathbf{r}) = V(r)$ be a spherically symmetric potential so that the Hamiltonian is

$$\hat{H} = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V(r). \tag{1}$$

Write the $\hat{\mathbf{L}}^2$ and $\hat{L}_z$ operators in differential form using a (θ, ϕ)-representation and prove that

10 pts $[\hat{H}, \hat{\mathbf{L}}^2] = 0$,

10 pts $[\hat{H}, \hat{L}_z] = 0$.

II. ACTION OF AM LADDER OPERATORS [20 pts]

[GR2 4.18] Let $|l, m\rangle$ be simultaneous eigenkets of $\mathbf{L}^2$ and L_z corresponding to eigenvalues $l(l+1)\hbar^2$ and $m\hbar$, respectively. Knowing that

$$L^{\pm}|l, m\rangle = C_{lm}|l, m \pm 1\rangle, \tag{2}$$

provided that $m \pm 1$ does not exceed its allowed values, calculate the normalization constant C_{lm} that ensures $\langle l, m|l, m\rangle = \langle l, m \pm 1|l, m \pm 1\rangle = 1$.