Quantum Physics 142 and 204.6

Problem Set 2

National Institute of Physics

Deadline: 27 November 2020

I. PERTURBATION I [40 pts]

Let a particle in an infinite square well of width a and centered at the origin be perturbed by a weakly linear potential αx.

10 pts Calculate the first-order energy correction to the ground state energy.

10 pts Calculate the first-order energy correction to the first excited state energy.

20 pts Calculate the second-order energy correction to the ground state energy.

II. PERTURBATION II [20 pts]

Let a particle be in a harmonic potential of natural frequency ω. Let a repulsive delta perturbation $\alpha\delta(x)$ be located at the enter of the harmonic potential.

20 pts Calculate the energy correction to the ground state energy up to second-order perturbation theory.

III. PERTURBATION III [20 pts]

Consider a one-electron atom in a uniform magnetic field **B** so that the perturbed Hamiltonian is

$$H' = \frac{e}{2m}(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}. \tag{1}$$

20 pts Show that in the weak Zeeman effect a spin-degenerate energy level splits into two levels labeled by m_j (the quantum number for the total angular momentum projection $J_z = L_z + S_z$) and that the energy splitting is linear with the field amplitude.

IV. VARIATIONAL ESTIMATE [20 pts]

Estimate the ground state energy of a hydrogen atom (Coulomb potential) using a Gaussian trial wavefunction $Ae^{-\alpha r^2}$. How far is this estimate from the exact value?